Random walking of a point mass on an oscillating plate

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In what follows, we show that models with simple randomized momentum impacts, even at their theoretical upper limit, would also severely under-estimate the persistence and speed of the ratcheting.

As a preliminary analysis, we calculate and show in Fig. 1(a) the time trajectory of a single particle taking off from a flat vibrating substrate, where we use a sinusoidal oscillation $(\Gamma g/\omega^2)\cos\omega t$ with the normalized coordinates $\phi \equiv \omega t$ and $\tilde{z} \equiv z/(\frac{\Gamma g}{\omega^2})$ to eliminate the explicit dependence on frequency. The normalized velocity of impact upon landing, $\tilde{v}_{rel}$, is also numerically solved as a function $f(\Gamma)$. As a first approximation, we start with simulating a single random walker in a spatially dependent excitation $\Gamma(x)$. Stochastic kicks are introduced with the assumptions that (a) for each cycle the particle takes off from the substrate vertically, that (b) the first landing impact gives the particle a random horizontal momentum $(\hat{x} \cdot \vec{I}) \sim \epsilon_{xz}\, m v_{rel}\, \xi$, and that (c) this momentum allows the particle to move along x for a characteristic time $\delta t \sim 2e\, v_{rel}/g$ as the energy is dissipated by subsequent impacts[1]. As a result, the particle would move by a distance $\delta x \sim \frac{1}{m}(\hat{x} \cdot \vec{I})\delta t = 2\epsilon|\Gamma f(\Gamma)|^2 \xi \frac{g}{\omega^2}$ in each cycle with $\epsilon \equiv \epsilon_{xz} e$, and the migration is determined by $x(t) = \int^t \delta x$. We also compare the results to simulations with a summation of uncorrelated kicks along a linear array of N particles so that an additional effect from the spatial gradient over the length-scale of Ns is taken into account[2], but find no significant difference upto N=8. Sample results are displayed in Fig. 1(b) with the $\Gamma(x)$ matching one of the experimental conditions, along with our laboratory data. Two fea-

[1] Here ξ represents a stochastic variable generated from a Gaussian distribution with a width of unity, m the particle mass, $v_{rel} = \tilde{v}_{rel}(\Gamma)\frac{\Gamma g}{\omega}$ the impact velocity, ϵ_{xz} a coupling constant reflecting the roughness of the substrate, and e the restitution coefficient.

[2] We take the approximation that N particles are constrained along x with a fixed spacing s, and replace the horizontal impulse $(\hat{x} \cdot \vec{I})$ with $\Sigma_i^N (\hat{x} \cdot \vec{I}_i)$, the stochastic variable ξ with ξ_i that are uncorrelated, and the time scale δt with a value evaluated at the center of mass.

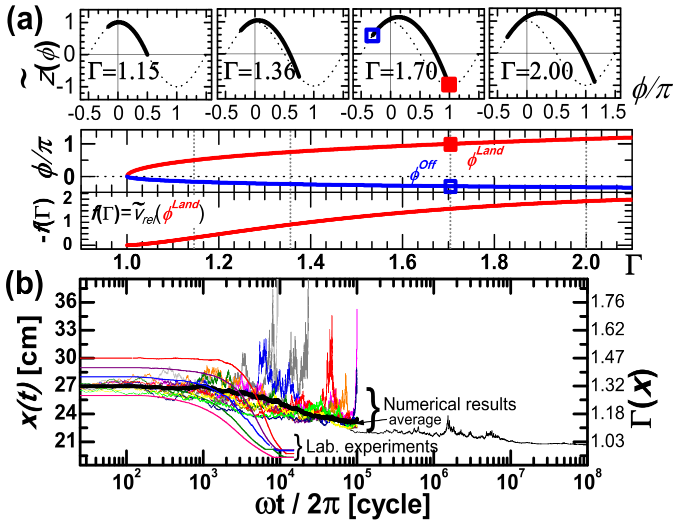

Figure 1: **(a)** Local analyses of one particle taking off from an oscillating substrate at various values of Γ, and the impact speeds upon landing; **(b)** Simulations with a spatial gradient: the numerical results shown here include a random selection of twelve individual runs, the ensemble average over 74 runs (up to 10^5cycle), and one run up to 10^8 cycles, for a point-like particle starting at $x(0) = 27$cm, with $\epsilon = 0.5$. The data from laboratory experiments with five initial conditions are also displayed here – note the relatively much smoother and faster motion of the granular chains in the experiments, compared to the numerical results.

tures are notable: (1) The numerical results exhibit prominent back-and-forth fluctuations[3] as opposed to the relatively smooth motion observed from laboratory experiments, even though the average does show a trend of going against the gradient. (2) The overall efficiency of the numerical migration is decades below that of the actual chains in experiments, even though the strength of our numerical kicks is already set at their theoretical upper limit by using $\epsilon \sim O(1)$. Namely, the simple "thermophoretic" modelling based on randomized impulses is inadequate to account for this highly efficient ratcheting against the gradient, with or without the summation along the chain.

We believe that inter-particle couplings, through the small phase lags, should be a key element in a minimal model for this lowest mode. Small *cooperative* movements among the constrained spheres– despite the anticipated increments in x being in microns that are perhaps below the accuracy of most particle tracking techniques– should have played crucial roles in creating coherent steps of crawling, thus outperforming the accumulation of uncorrelated random walks.

[3]The fluctuations in the simulations would not be surprising if one imagines the object "detecting" the gradient primarily by visiting different locations, in resemblance to random walks of small molecules or micro-organisms in nature.